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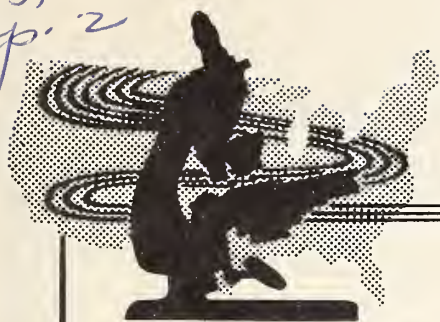
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STORAGE FLOOD ROUTING WITHOUT COEFFICIENTS

UNITED STATES DEPARTMENT OF AGRICULTURE

CONTENTS

STORAGE FLOOD ROUTING WITHOUT COEFFICIENTS

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INTRODUCTION

Flood routing methods that are based on the conservation of mass (continuity) and a single-valued relationship between depth of flow and flow rate are usually termed "storage flood routing" methods. This type of formulation is discussed more rigorously under the theory of kinematic waves. A fundamental reference to this theory is made by Lighthill and Whitham (7).[2] Henderson (4) considered the kinematic treatment of flood waves in prismatic channels. More recently, Henderson and Wooding (5) applied the kinematic wave theory to the overland flow problem.

Many hydrologists using storage flood routing utilize coefficients, such as X and K of the Muskingum method (2) or C of the convex method.[3] This report presents a numerical method for storage flood routing without coefficients. A number of flow problems are solved with a computer program. Some limitations of the kinematic formulation are illustrated by example.

FORMULATION

The formulation of storage of kinematic flood routing is composed of the continuity equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \tag{1}$$

and a rating function

$$Q = Q(A) \tag{2}$$

where

$\quad Q$ = flow rate, c.f.s.,

$\quad A$ = flow area, ft.^{2},

$\quad q$ = lateral inflow (+) or lateral outflow (−), c.f.s./ft., and

$\quad x, t$ = distance (feet) and time (seconds) coordinates.

The salient features of the continuity equations are obtained by substituting $Q = AV$ into equation 1, differentiating the product, and multiplying both sides by $\Delta t \Delta x$, to obtain volumes,

$$\Delta x \Delta t \left(\overset{(a)}{\frac{A \partial V}{\partial x}} + \overset{(b)}{\frac{V \partial A}{\partial x}} + \overset{(c)}{\frac{\partial A}{\partial t}} \right) = q (\Delta x \Delta t).$$

where

$$V = \text{velocity, ft./sec.}$$
$$\Delta t = \text{time increment, sec.}$$
$$\Delta x = \text{length increments, ft.}$$

The terms on the left-hand side are usually referred to as follows:

(a) - prism storage,
(b) - wedge storage, and
(c) - time rate of change of storage.

The usual treatment of equations 1 and 2 is to recast them as follows:

Using the grid pattern in figure 1, equation 1 can be written (with $q = 0$),
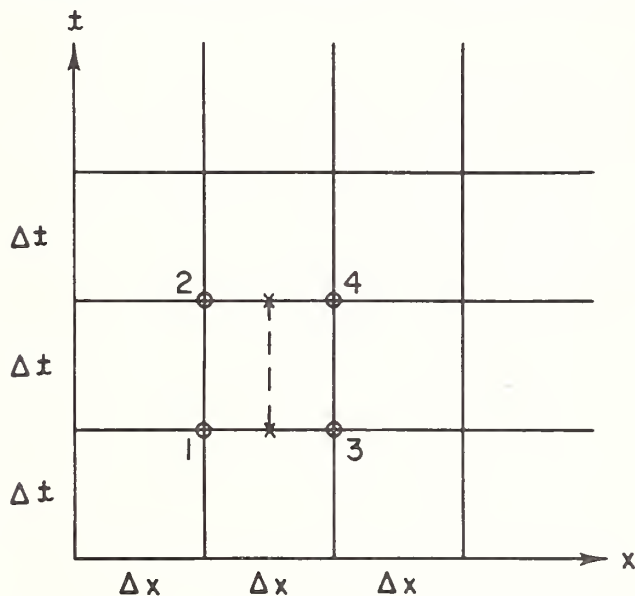
Figure 1.--Grid for finite difference approximations.

$$\frac{\partial Q}{\partial x} = \frac{Q_4 - Q_2}{\Delta x}$$

$$\frac{\partial A}{\partial t} = \frac{A_4 - A_3 + A_2 - A_1}{2 \Delta t}$$

$$\bar{q} = \frac{q_4 + q_2}{2}$$

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$$\frac{Q_4 - Q_2}{\Delta x} + \frac{(A_4 + A_2) - (A_3 + A_1)}{2\Delta t} = 0$$

or

$$Q_4 - Q_2 + \left(\frac{(A_4 + A_2)}{2}\Delta x - \frac{(A_3 + A_1)}{2}\Delta x\right)\frac{1}{\Delta t} = 0. \tag{3a}$$

Now if we define

$$S_2 = \left(\frac{(A_4 + A_2)}{2}\right)\Delta x = \text{storage at } t = t + \Delta t,$$

$$S_1 = \frac{(A_3 + A_1)}{2}\Delta x = \text{storage at } t = t,$$

$$Q_4 = 0 = \text{outflow, and}$$

$$Q_2 = I = \text{inflow,}$$

The equation 3a can be written

$$I - 0 = \frac{S_2 - S_1}{\Delta t}$$

or

$$(I - 0)\Delta t = \Delta S. \tag{3b}$$

Equation 3b is similar to the usual continuity expression (inflow - outflow = change of storage). Storage is evaluated as

$$S = S(0) \tag{4}$$

or

$$S = S(0, I) \tag{5}$$

where

$$S = \text{storage, c.f.s.-hrs.,}$$
$$0 = \text{outflow, c.f.s., and}$$
$$I = \text{inflow, c.f.s.}$$

Equation 4 is used by methods that neglect wedge storage, such as the storage-indication method. Equation 5 is used by the Muskingum method where two coefficients X and K are introduced to account for both prism and wedge storage. The development in this report is based on only equations 1 and 2, i.e., no functional form is assumed for relating reach storage to inflow and outflow. The assumption inherent in kinematic wave theory is not altered; i.e., the equation of motion can be approximated by a single-valued function, equation 2.

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SOLUTION FORMULATION

A simultaneous solution of equations 1 and 2 is accomplished by obtaining a numerical solution on a small computer (IBM 1620, 40 K memory and indirect addressing). Equation 1 is approximated by the finite difference quotients shown in figure 1. Substituting these quotients into equation 1, one obtains:

$$\frac{Q_4 - Q_2}{\triangle x} + \frac{A_4 - A_3 + A_2 - A_1}{2\triangle t} = \bar{q}$$

or

$$\frac{\triangle t}{\triangle x} Q_4 + \frac{A_4}{2} = \left(\frac{A_1 + A_3}{2}\right) + \frac{\triangle t}{\triangle x} Q_2 + \left(\frac{-A_2}{2} + \triangle t\bar{q}\right)$$

or

$$\lambda Q_4 + \frac{A_4}{2} = \alpha + \beta \tag{6}$$

where

$$\lambda = \triangle t/\triangle x,$$

$$\alpha = (A_1 + A_3)/2, \text{ and}$$

$$\beta = \lambda Q_2 + \left(\frac{-A_2}{2} + \triangle t\bar{q}\right).$$

The subscripts in these equations are defined as follows:

A_1 = flow area at location x and time t,

A_3 = flow area at location x $+\triangle$x and time t,

A_2, Q_2 = flow area or rate at location x and time t $+ \triangle$t, and

A_4, Q_4 = flow area or rate at location x $+\triangle$x and time t $+ \triangle$t.

Equation 2 is retained in the computer memory as an array of numerical values, i.e., paired Q and A values.

The rating functions are determined by one of several procedures. If normal flow (turbulent flow) is assumed, the Manning or Chezy equation can be applied at each channel section. An alternative for natural channels is to calculate a series of water surface profiles; e.g., using U.S. Department of Agriculture Hydrograph Laboratory Computer Program No. 2 (8). The computed values for rates of flow and depths of flow at each section along the channel system define a section rating function.

Since the rating function is not an explicit function but a set of numerical values, a simultaneous solution of equations 6 and 2 requires an interation procedure. In this study, the procedure used is known as the method of false position 6. A special constraint is required to account for incipient flow at downstream sections during rising stages and for cessation of flow at upstream sections during falling stages, i.e., for value of ($\alpha + \beta$) $\leq$ 0, Q_4 and A_4 are set equal to base flow.

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SOLUTION PROGRAM

Presented in appendix 1 are the source programs for the main line and three subroutines. These programs are written in Fortran II.

Program input and output formats are presented in appendix 2. Arrays of data are read in by way of the "Read" subroutine. Flow area values are calculated for flow rate values by an interpolation function. Arguments are obtained from the "Table Look-up" subroutine. One limitation on inflow input is that the final rate of inflow must equal the initial value (return to a dry channel or to the initial base flow). Note that lateral flow hydrographs must have the same number of entries as the inflow hydrograph. Dummy zeros can be entered.

In storage flood routing methods, calculations proceed, as usual, in a downstream direction. An inflow hydrograph is given at the upstream section as a boundary condition. For overland flow applications, this is the "null" hydrograph. The initial conditions are known rates of flow and flow areas. These can be zero if a dry channel is assumed. If base flow is assumed, flow area should be calculated from the section rating functions. Lateral flow (q) can be put in as a plus quantity for local inflows or as a minus quantity for transmission losses. Inflow from a tributary can be treated as lateral flow introduced over a short channel length.

Initially the program section, "Routing During Inflow," determines the time of incipent flow at the next downstream section; thereafter, inflow is routed by application of the "Solve" subroutine. This subroutine performs the iteration solution of equation 6 with discharge values calculated from the rating function table. Iteration cycling is terminated by a tolerance (TOLR) level.

After the last value of inflow has been routed, computations are transferred to the main line program section "Routing After Inflow" section of the program. Routing computations continue until downstream outflow decreases to a value within 0.1 of the initial value; the last outflow entry is then set equal to the initial inflow value. All input data arrays are filled out to correspond in length to the outflow arrays. During solution punch out, outflow section data are interchanged with inflow section data so as to become inflow data for the next routing reach. After data arrays are read in for the next outflow section, computations proceed for the next routing reach. Problem solution is complete after the last section data have been read in and a Reader "no feed" signal occurs on the IBM 1620 console.

EXAMPLES

Example 1 is an inflow hydrograph introduced on a constant base flow of 150 c.f.s. Manning's equation was utilized for the rating function of a theoretical rectangular channel 30 feet wide, with a bed slope of 0.001 and an n-value of 0.03. Since this was a prismatic channel, the rating functions are the same at all sections of the channel. The time scale increment ($\triangle t$) was 3,600 seconds, and the distance scale increment ($\triangle x$) was 7,200 feet. Table 1 presents program input and table 2 presents the program output. Figure 2 is a succession of routed hydrographs. Figure 3 is a typical storage outflow function generated by the routing method. The storage values were calculated by the average end-area method. The storage loop indicates the importance of wedge storage. If the wedge storage term in the continuity equation were neglected, the relationship would be a single-valued curve.

For example 2, an inflow hydrograph was introduced on an initially dry channel with a constant (in time and space) lateral outflow (q) of 0.05 c.f.s./ft. The same channel geometry was used as in example 1, and Manning's equation was used to calculate the rating functions. The program input is presented in table 3. A succession of routed hydrographs is presented in figure 4. The rather uniform decline in the hydrograph results from the constant transmission loss.

Figure 5 presents two additional surface flow problems. Presented in figure 5 at top, is a set of hydrographs resulting from a constant inflow of 150 c.f.s. This problem would be typical of the border of furrow irrigation problem. Also, releases from a detention reservoir might lead to a similar problem. In this example, transmission losses were set to zero. The program input for this example is given in table 4. Figure 5, at bottom, illustrates the overland flow

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Table 1.--Input for example 1

```
3600 3600 50   50    1   27 35                                              001
       0      1      2        4        7       11       16       23       30 1 1 00
      40     50     60       80      100      120      150      200      250 2 1 00
     300    350    400      450      500      550      600      650      700 3 1 00
     800    900   1000     1100     1200     1300     1400     1500          4 100
       0   .1633  .5179   1.6369   4.1364   8.7595  16.2298  29.4354  45.4092 1 1 11
 72.3008 103.49  138.41   217.91   308.26   408.21   572.12   875.32  1209.9  2 1 11
1565.15 1939.71 2325.76  2724.55  3133.9   3548.1   3970.2   4397.9   4831.0  3 1 11
 5709.5   6601   7503     8408     9326    10245    11177    12103          4 1 11
     150    400    650     1088     1775     2775     3775     4275     4400 1 1 22
    4275   3838   3338     2775     2338     1963     1650     1338     1088 2 1 22
     900    713    525      400      275      150      150      150      150 3 1 22
       0      0      0       -0        0        0        0        0        0 3 1 22
       0      0      0        0        0        0        0        0        0 2 1 22
      -0      0      0        0        0        0        0        0        0 1 1 22
       0      1      2        4        7       11       16       23       30 1 1 00
      40     50     60       80      100      120      150      200      250 2 1 00
     300    350    400      450      500      550      600      650      700 3 1 00
     800    900   1000     1100     1200     1300     1400     1500          4 100
       0   .1633  .5179   1.6369   4.1364   8.7595  16.2298  29.4354  45.4092 1 1 11
 72.3008 103.49  138.41   217.91   308.26   408.21   572.12   875.32  1209.9  2 1 11
1565.15 1939.71 2325.76  2724.55  3133.9   3548.1   3970.2   4397.9   4831.0  3 1 11
 5709.5   6601   7503     8408     9326    10245    11177    12103          4 1 11
       0      0      0        0        0        0        0        0        0 1 1 44
       0      0      0        0        0        0        0        0        0 2 1 44
       0      0      0        0        0        0        0        0        0 3 1 44
    7200    150                                                              002
```


SUCCESSION OF HYDROGRAPHS WITH A
BASE FLOW OF 150 CFS

INFLOW

OUTFLOW
57,600 FEET
DOWNSTREAM

Figure 2.--Succession of routed hydrographs.

Table 2.--Output for example 1

IN SECTION NO = 1		OUT SECTION NO = 2	
IN AREA	IN DISCH	OUT AREA	OUT DISCH
62.915	150.000	62.915	150.000
118.357	400.000	99.849	307.579
162.843	650.000	147.341	557.596
231.783	1088.000	211.841	954.559
328.012	1775.000	302.998	1587.613
456.162	2775.000	424.965	2524.881
576.877	3775.000	547.966	3531.256
635.632	4275.000	620.306	4143.905
650.242	4400.000	645.582	4360.114
635.632	4275.000	638.202	4296.988
584.340	3838.000	595.476	3932.010
524.637	3338.000	538.697	3454.469
456.162	2775.000	472.599	2909.573
401.534	2338.000	415.538	2449.689
353.016	1963.000	365.748	2061.307
311.326	1650.000	322.645	1734.789
268.029	1338.000	280.105	1423.800
231.783	1088.000	242.763	1161.472
203.688	900.000	212.420	958.435
173.232	713.000	183.072	772.670
141.375	525.000	152.521	587.407
118.357	400.000	127.306	448.132
92.637	275.000	103.261	324.558
62.915	150.000	76.982	205.916
62.915	150.000	65.720	161.147
62.915	150.000	63.475	152.223
62.915	150.000	63.027	150.443
62.915	150.000	62.915	150.000

IN SECTION NO = 2		OUT SECTION NO = 3	
IN AREA	IN DISCH	OUT AREA	OUT DISCH
62.915	150.000	62.915	150.000
99.849	307.579	86.461	247.099
147.341	557.596	130.575	465.990
211.841	954.559	192.227	828.188
302.998	1587.613	278.227	1410.453
424.965	2524.881	394.877	2286.205
547.966	3531.256	518.228	3284.904
620.306	4143.905	602.050	3987.742
645.582	4360.114	638.380	4298.508
638.202	4296.988	638.993	4303.754
595.476	3932.010	604.556	4009.174
538.697	3454.469	551.875	3563.929
472.599	2909.573	488.422	3039.118
415.538	2449.689	430.014	2565.149
365.748	2061.307	378.825	2162.272
322.645	1734.789	334.335	1822.364

Table 2.--Output for example 1--Continued

280.105	1423.800	292.033	1508.545
242.763	1161.472	253.818	1237.030
212.420	958.435	221.745	1020.835
183.072	772.670	192.665	830.846
152.521	587.407	162.528	648.094
127.306	448.132	136.630	499.075
103.261	324.558	112.799	372.226
76.982	205.916	88.473	256.190
65.720	161.147	72.484	188.034
63.475	152.223	65.718	161.139
63.027	150.443	63.653	152.931
62.915	150.000	63.085	150.673
62.915	150.000	62.949	150.134
62.915	150.000	62.915	150.000

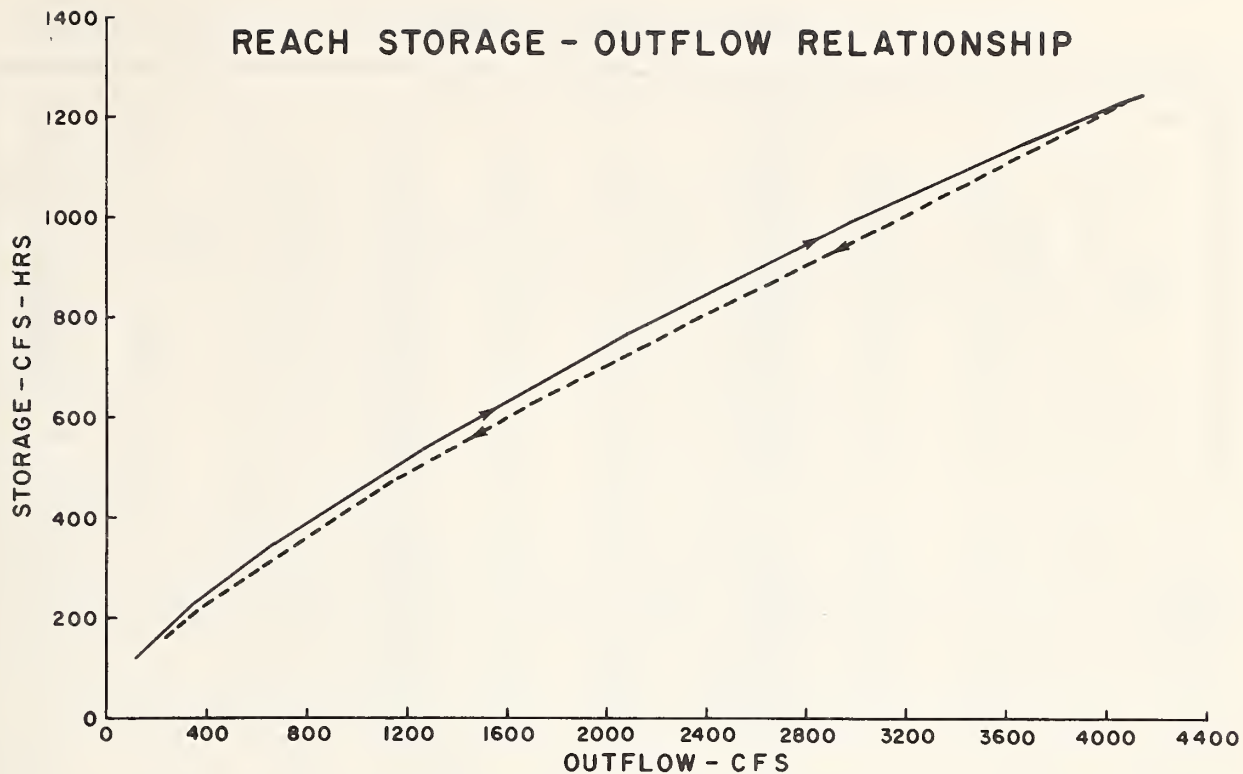| IN SECTION NO = 3		OUT SECTION NO = 4	
IN AREA	IN DISCH	OUT AREA	OUT DISCH
62.915	150.000	62.915	150.000
86.461	247.099	77.694	208.744
130.575	465.990	115.222	384.332
192.227	828.188	172.598	709.159
278.227	1410.453	254.583	1242.469
394.877	2286.205	365.414	2058.725
518.228	3284.904	488.359	3038.602
602.050	3987.742	581.156	3811.123
638.380	4298.508	628.588	4214.746
638.993	4303.754	637.840	4293.889
604.556	4009.174	611.644	4069.808
551.875	3563.929	563.784	3664.470
488.422	3039.118	503.495	3162.859
430.014	2565.149	444.704	2682.313
378.825	2162.272	392.244	2265.878
334.335	1822.364	346.389	1912.664
292.033	1508.545	303.778	1593.458
253.818	1237.030	264.697	1314.324
221.745	1020.835	231.493	1086.063
192.665	830.846	202.093	889.326
162.528	648.094	172.390	707.893
136.630	499.075	146.123	550.939
112.799	372.226	122.143	419.922
88.473	256.190	98.883	303.215
72.484	188.034	80.855	221.774
65.718	161.139	70.075	178.460
63.653	152.931	65.344	159.655
63.085	150.673	63.648	152.914
62.949	150.134	63.116	150.796
62.915	150.000	62.962	150.185
62.915	150.000	62.915	150.000

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Figure 3.--Reach storage-outflow relationship.

Figure 4.--Succession of routed hydrographs with a constant transmission loss.
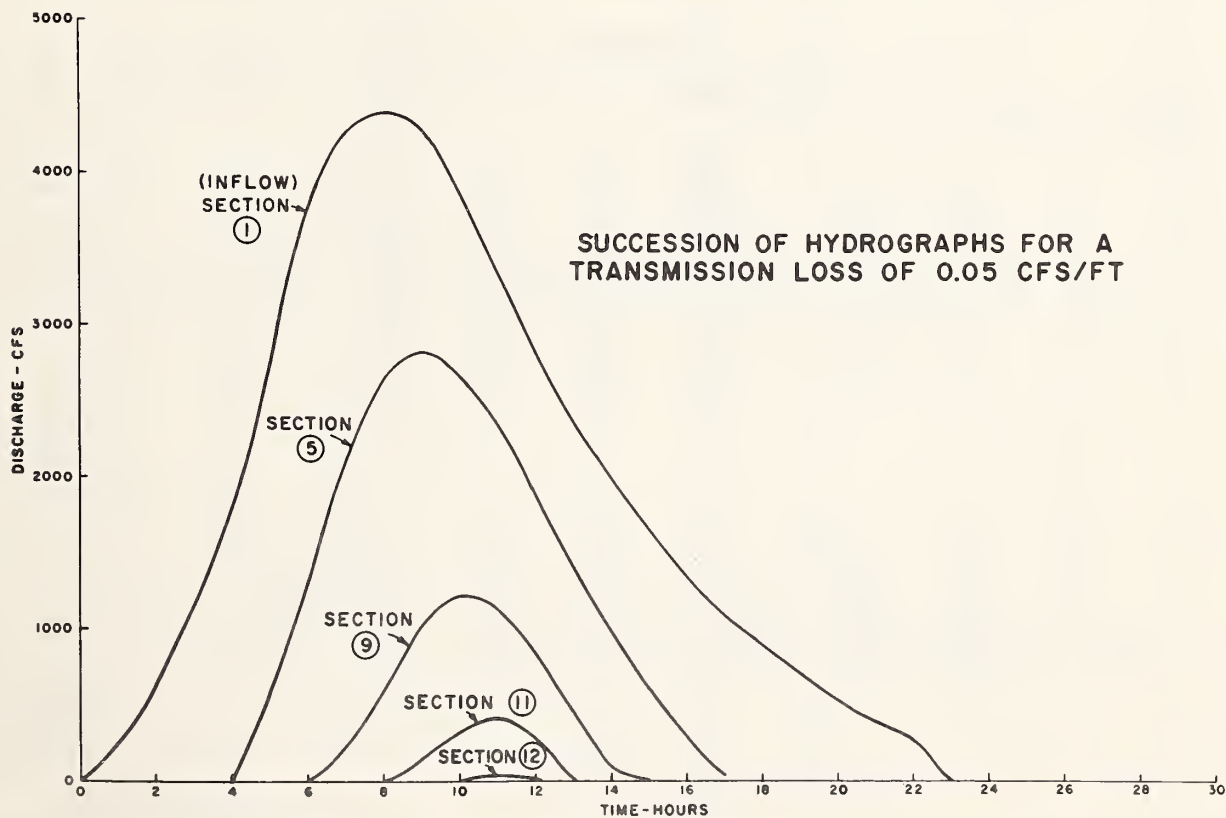
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Table 3.--Input for example 2

```
3600 3600 50   50    1  27 35                                                    001
   0        1        2        4        7       11       16       23       30 1 1 00
  40       50       60       80      100      120      150      200      250 2 1 00
 300      350      400      450      500      550      600      650      700 3 1 00
 800      900     1000     1100     1200     1300     1400     1500          4 100
   0     .1633    .5179   1.6369   4.1364   8.7595  16.2298  29.4354  45.4092 1 1 11
 72.3008 103.49   138.41   217.91   308.26   408.21   572.12   875.32  1209.9 2 1 11
1565.15 1939.71  2325.76  2724.55   3133.9   3548.1   3970.2   4397.9  4831.0 3 1 11
5709.5   6601     7503     8408     9326    10245    11177    12103          4 1 11
   0      250      650     1188     1775     2775     3775     4275     4400 1 1 22
4275     3838     3338     2775     2338     1963     1650     1338     1088 2 1 22
 900      713      525      400       27        0        0        0        0 3 1 22
 -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05
 -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05
 -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05
   0        1        2        4        7       11       16       23       30 1 1 00
  40       50       60       80      100      120      150      200      250 2 1 00
 300      350      400      450      500      550      600      650      700 3 1 00
 800      900     1000     1100     1200     1300     1400     1500          4 100
   0     .1633    .5179   1.6369   4.1364   8.7595  16.2298  29.4354  45.4092 1 1 11
 72.3008 103.49   138.41   217.91   308.26   408.21   572.12   875.32  1209.9 2 1 11
1565.15 1939.71  2325.76  2724.55   3133.9   3548.1   3970.2   4397.9  4831.0 3 1 11
5709.5   6601     7503     8408     9326    10245    11177    12103          4 1 11
 -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05
 -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05
 -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05     -.05
7200       0                                                                    002
```

Table 4.--Input for example shown in figure 5, at top

```
3600 3600 50   50    1  27 35                                                    001
   0        1        2        4        7       11       16       23       30 1 1 00
  40       50       60       80      100      120      150      200      250 2 1 00
 300      350      400      450      500      550      600      650      700 3 1 00
 800      900     1000     1100     1200     1300     1400     1500          4 100
   0     .1633    .5179   1.6369   4.1364   8.7595  16.2298  29.4354  45.4092 1 1 11
 72.3008 103.49   138.41   217.91   308.26   408.21   572.12   875.32  1209.9 2 1 11
1565.15 1939.71  2325.76  2724.55   3133.9   3548.1   3970.2   4397.9  4831.0 3 1 11
5709.5   6601     7503     8408     9326    10245    11177    12103          4 1 11
   0      150      150      150      150      150      150      150      150
 150      150      150      150      150      150      150      150      150
 150      150      150      150      150      150      150      150        0
   0        0        0        0        0        0        0        0        0 1 1 22
   0        0        0        0        0        0        0        0        0 2 1 22
   0        0        0        0        0        0        0        0        0 3 1 22
   0        1        2        4        7       11       16       23       30 1 1 00
  40       50       60       80      100      120      150      200      250 2 1 00
 300      350      400      450      500      550      600      650      700 3 1 00
 800      900     1000     1100     1200     1300     1400     1500          4 100
   0     .1633   . .5179   1.6369   4.1364   8.7595  16.2298  29.4354  45.4092 1 1 11
 72.3008 103.49   138.41   217.91   308.26   408.21   572.12   875.32  1209.9 2 1 11
1565.15 1939.71  2325.76  2724.55   3133.9   3548.1   3970.2   4397.9  4831.0 3 1 11
5709.5   6601     7503     8408     9326    10245    11177    12103          4 1 11
   0        0        0        0        0        0        0        0        0 1 1 22
   0        0        0        0        0        0        0        0        0 2 1 22
   0        0        0        0        0        0        0        0        0 3 1 22
7200       0                                                                    002
```
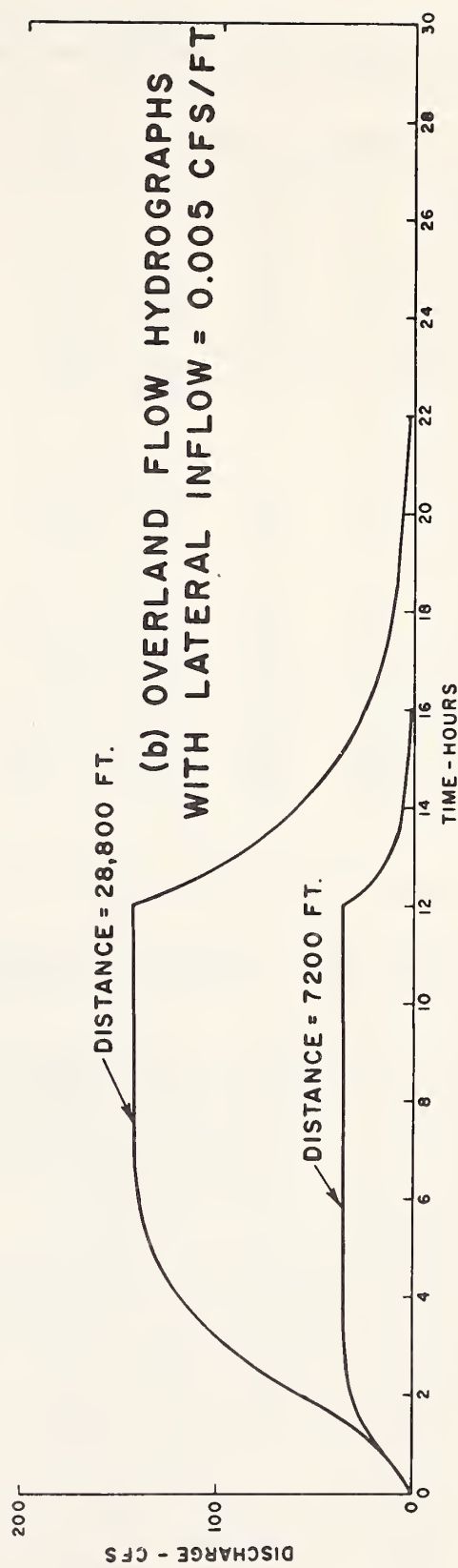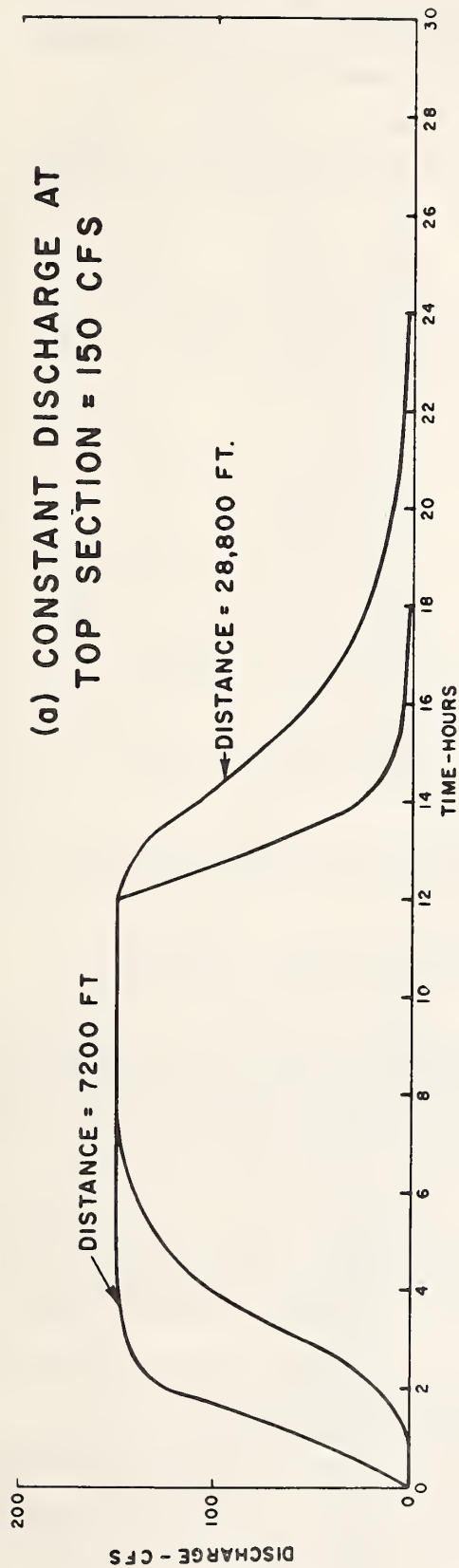
Figure 5.—Top, Succession of hydrographs with a constant inflow at top section; Bottom, Succession of overland flow hydrographs.

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problem. Inflow is a uniform and constant lateral rate of 0.005 c.f.s./ft. The rate could be assumed to be falling over a watershed and could be varied so as to evaluate the influence of temporal and spatial variations of rainfall. Program input is presented in table 5.

Table 5.--Input for example shown in figure 5, at bottom

```
3600 3600 50   50    1   27  35                                                   001
       0        1        2        4        7       11       16       23       30 1 1 00
      40       50       60       80      100      120      150      200      250 2 1 00
     300      350      400      450      500      550      600      650      700 3 1 00
     800      900     1000     1100     1200     1300     1400     1500          4 100
       0    .1633    .5179   1.6369   4.1364   8.7595  16.2298  29.4354  45.4092 1 1 11
 72.3008   103.49   138.41   217.91   308.26   408.21   572.12   875.32   1209.9 2 1 11
 1565.15  1939.71  2325.76  2724.55   3133.9   3548.1   3970.2   4397.9   4831.0 3 1 11
  5709.5     6601     7503     8408     9326    10245    11177    12103          4 1 11
      .0        0        0        0        0        0        0        0        0 1 1 22
       0        0        0        0        0        0        0        0        0 2 1 22
       0        0        0        0        0        0        0        0        0 3 1 22
    .005     .005     .005     .005     .005     .005     .005     .005     .005
    .005     .005     .005     .005     .005     .005     .005     .005     .005
    .005     .005     .005     .005     .005     .005     .005     .005        0
       0        1        2        4        7       11       16       23       30 1 1 00
      40       50       60       80      100      120      150      200      250 2 1 00
     300      350      400      450      500      550      600      650      700 3 1 00
     800      900     1000     1100     1200     1300     1400     1500          4 100
       0    .1633    .5179   1.6369   4.1364   8.7595  16.2298  29.4354  45.4092 1 1 11
 72.3008   103.49   138.41   217.91   308.26   408.21   572.12   875.32   1209.9 2 1 11
 1565.15  1939.71  2325.76  2724.55   3133.9   3548.1   3970.2   4397.9   4831.0 3 1 11
  5709.5     6601     7503     8408     9326    10245    11177    12103          4 1 11
    .005     .005     .005     .005     .005     .005     .005     .005     .005
    .005     .005     .005     .005     .005     .005     .005     .005     .005
    .005     .005     .005     .005     .005     .005     .005     .005        0
 7200        0                                                                    002
```

CONSISTENCY OF THE STORAGE FLOOD ROUTING METHOD

A vexing characteristic of storage flood routing methods is the dependence of calculated discharges on the selection of reach lengths ($\triangle x$) and time increments ($\triangle t$). An indication of this difficulty with respect to storage flood routing with coefficients Muskingum was discussed by Gilcrest 3, thus: "However, it has been found that actual flood hydrographs along several hundred miles of river can, through judicious selection of routing reaches and time interval used in routing, be duplicated quite accurately...."

Several numerical trials were made to evaluate the influence of $\triangle x$ and $\triangle t$ on the storage flood routing without coefficients method. Tables 6 and 7 represent the routing of the same inflow hydrograph used in example 1. Three reach lengths are combined with two time increments. The indication is that the effect of $\triangle x$ over the range shown here for a constant $\triangle t$ is small. However, an influence of $\triangle t$ is clearly evident.

Table 8 presents a set of routings for a triangular-shaped hydrograph inflow into a prismatic channel. The conclusion is that the peak reduction is completely dependent on $\triangle t$.

This conclusion was first pointed out by Thomas (9). Especially evident from table 8 is a possible trend toward no crest reduction. This is in agreement with the work of Henderson (4) on kinematic waves. Thus, the method of storage flood routing is converging on the correct result. In view of the approach to a condition of no peak reduction as $\triangle t \longrightarrow 0$, an effort is required to select $\triangle t$ so as to approximate the true peak reduction condition. Presented in

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Table 6.--Comparison of hydrographs for a time increment of 1 hour
with three reach lengths

Time (hours)	Hydrograph discharge		
	$\Delta x = 3,600$ feet	$\Delta x = 7,200$ feet	$\Delta x = 14,400$ feet
	C.f.s.	C.f.s.	C.f.s.
0	150	150	150
2	183	182	179
4	560	560	561
6	1,866	1,866	1,867
8	3,502	3,503	3,507
10	4,117	4,117	4,118
12	3,665	3,665	3,663
14	2,804	2,804	2,802
16	2,038	2,038	2,038
18	1,443	1,443	1,443
20	1,001	1,001	1,001
22	667	667	667
24	419	419	420
26	263	263	261
28	187	187	187
30	158	159	159
32	151	151	152
34	150	150	150
36	150	150	150

Table 7.--Comparison of hydrographs for a time increment of one-half hour
with three reach lengths

Time (hours)	Hydrograph discharge		
	$\Delta x = 3,600$ feet	$\Delta x = 7,200$ feet	$\Delta x = 14,400$ feet
	C.f.s.	C.f.s.	C.f.s.
0	150	150	150
2	158	157	150
4	443	445	452
6	1,768	1,769	1,771
8	3,622	3,623	3,628
10	4,262	4,262	4,260
12	3,768	3,767	3,765
14	2,816	2,816	2,814
16	2,023	2,024	2,024
18	1,435	1,435	1,435
20	994	994	995
22	662	662	662
24	410	411	412
26	243	243	241
28	169	169	171
30	150	152	153
32	150	150	150
34	150	150	150
36	150	150	150

Table 8.--Comparison of hydrographs for several time intervals with a
constant reach length (prismatic channel)

Time (hours)	Inflow	Outflow 72,000 feet downstream for Δt of--			
		1/10 hour	1/4 hour	1/2 hour	1 hour
	C.f.s.	C.f.s.	C.f.s.	C.f.s.	C.f.s.
0	150	150	150	150	150
1/2	600	150	150	150	--
1	1,200	150	150	150	159
1 1/2	1,800	150	150	151	--
2	2,400	150	150	159	260
2 1/2	3,000	150	150	204	--
3	3,600	150	177	391	733
3 1/2	4,200	286	531	827	--
4	4,800	1,276	1,330	1,476	1,721
4 1/2	4,390	2,260	2,206	2,207	--
5	3,975	3,045	2,989	2,898	2,801
5 1/2	3,575	3,741	3,636	3,451	--
6	3,180	4,277	4,060	3,801	3,460
6 1/2	2,790	4,402	4,193	3,933	--
7	2,400	4,194	4,075	3,875	3,551

tables 9 and 10 are the comparisons of the selection of Δt in the storage flood routing procedure so as to correspond to the peak reduction found with a higher order routing method (1). A Δt in the range of one-fourth to one-half hour appears to give a satisfactory correspondence. Figures 6 and 7 show the tabular comparison between routed hydrographs for two different hydrograph shapes. In figure 7, the 1/4-hour hydrograph could not be shown since it fell too close to the hydrograph predicted by the higher order routing method.

CONCLUSIONS

Formulation of the unsteady, non-uniform open channel flow problem in pure kinematics leads to a simple problem for solution. The computer program listed here appears to be functioning quite well over a wide spectrum of problems.

An unavoidable feature of storage flood routing methods is still of concern; namely, the selection of the reach length (Δx) and time increment (Δt). Tentative results of the numerical testing in this report indicate that the only concern in selecting the reach length is to fit the physical dimension of the channel system. However, the time increment selection has major influence on the shape of the routed hydrograph. For two extreme inflow hydrograph shapes, several time increment comparisons were made with a higher order routing method, i.e., a more complete equation of motion was utilized. The best correspondence occurred when the time increment was $\Delta t = tp/16$

where

tp = time to peak of the inflow graph.

Future development for this routing method will require extensive testing with data from actual floodflow situations. It may happen that the strongly nonprismatic character of actual channels may reduce the influence of time increments.

Table 9.--Comparison of storage flood routing read in 1/2-hour time increment
without coefficients with a higher order formulation

Time (hours)	Inflow	Outflow[1]	Outflow[2]	
			$\Delta t = 1/4$ hour	$\Delta t = 1/2$ hour
	C.f.s.	C.f.s.	C.f.s.	C.f.s.
0	150	150	150	150
1/2	600	150	150	150
1	1,200	150	150	150
1 1/2	1,800	150	150	151
2	2,400	150	150	159
2 1/2	3,000	150	150	204
3	3,600	159	177	391
3 1/2	4,200	390	531	827
4	4,800	1,380	1,330	1,476
4 1/2	4,390	2,455	2,206	2,207
5	3,980	3,126	2,989	2,898
5 1/2	3,580	3,703	3,636	3,451
6	3,180	4,002	4,060	3,801
6 1/2	2,790	4,062	4,193	3,933
7	2,400	3,923	4,075	3,875

[1] Calculated with the equation of motion, neglecting only the inertia forces.
[2] Calculated by storage flood routing without coefficients.

Table 10.--Comparison of storage flood routing read in hourly time increments
without coefficients with a higher order formulation

Time (hours)	Inflow	Outflow[1]	Outflow[2]		
			$\Delta t = 1/4$ hour	$\Delta t = 1/2$ hour	$\Delta t = 1$ hour
	C.f.s.	C.f.s.	C.f.s.	C.f.s.	C.f.s.
0	150	150	150	150	150
1	400	154	150	150	150
2	650	157	150	157	182
3	1,075	182	150	213	288
4	1,775	349	380	445	559
5	2,775	921	910	949	1,082
6	3,775	1,810	1,730	1,769	1,866
7	4,275	2,837	2,790	2,767	2,753
8	4,400	3,737	3,700	3,623	3,503
9	4,275	4,176	4,190	4,119	3,970
10	3,875	4,274	4,320	4,262	4,117
11	3,375	4,101	4,150	4,119	3,988
12	2,775	3,742	3,800	3,767	3,665

[1] Calculated with the equation of motion, neglecting only the inertia forces.
[2] Calculated by storage flood routing without coefficients.
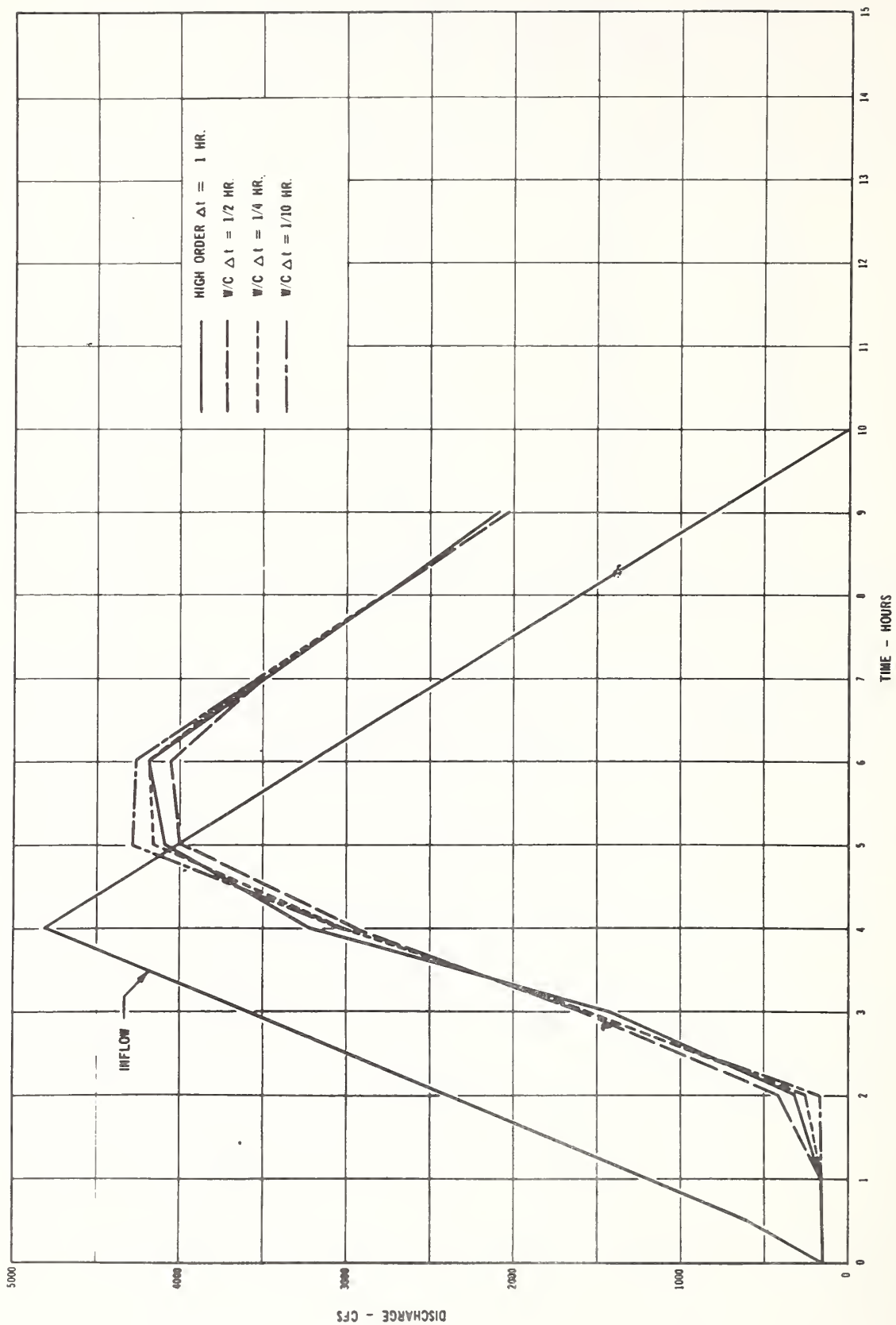
Figure 6.—Storage flood routing with different time increments compared with a higher-order formulation.

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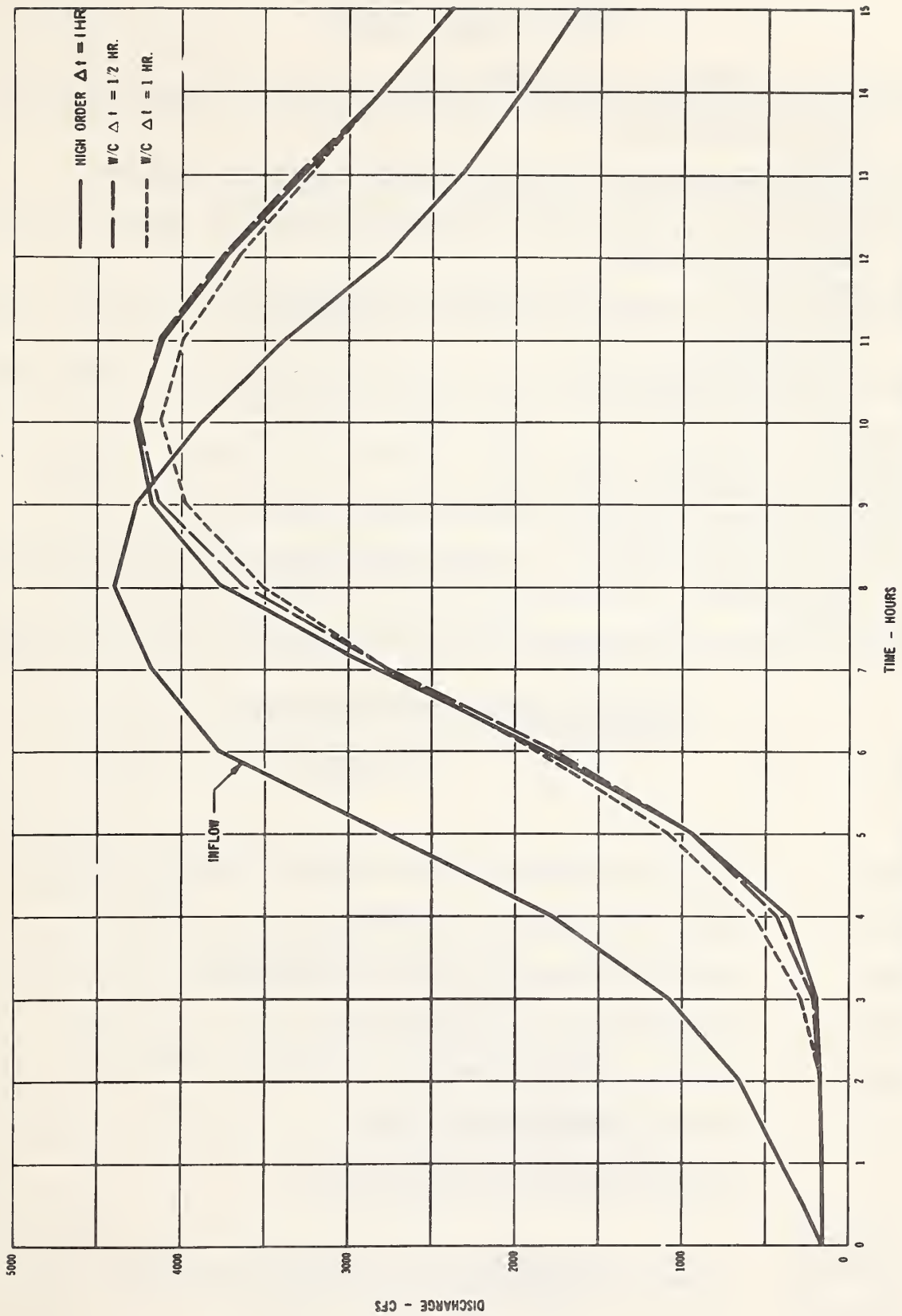
Figure 7.—Storage routing with different time increments compared with a higher-order formulation.

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SYMBOLS USED IN THE PROGRAM

Symbol	Description

Symbol — Description

Input:

Symbol	Description
DELT	Time increment used in routing computations (constant) in seconds
DELTI	Same as DELT, saved for initialization
DELA	Area increment in rating table (see Solve subroutine)
DELA1	Same as DELA, saved for initialization
TOLR	Tolerance (see Solve subroutine)
N1	Number of inflow hydrograph entries
N2	Number of tabulated rating function entries
AREA1(I) DISCH1(I) AREA2(I) DISCH2(I)	Entries in the upper and lower section rating function tables

Symbol	Description
(I = 1, N2)	
Q1 (I)	Entries in the inflow hydrograph
QL1(I)	
QL2(I)	Entries in the upper and lower lateral inflow hydrographs
(I = 1, N1)	
Q2(I)	Initial flow rate at the lower section
(I = 1)	
DELX	Distance increment (can be changed for each reach)

Output:

Symbol	Description
LL1	Upper section number
LL2	Lower section number
A1 (I)	
Q1 (I)	Upper section flow area and flow rate
A2 (I)	
Q2 (I)	Lower section flow area and flow rate
(I = 1, M)	

Main Program:

Symbol	Description
READ	See Read subroutine
TBLLP	See Table Look-up subroutine
XINT	Function for interpolation
(C, D, E, F, G)	Arithmetic statement arguments
	C – Lower bound for wanted table
	D – Upper bound for wanted table
	E – Lower bound for argument table
	F – Upper bound for argument table
	G – Argument
N11	Value of N1 saved for initialization
M	Total number of outflow entries
I	Index of do loop

Symbol	Description
J1	Lower index value of a do loop used for filling out lateral inflow hydrograph entries
J	Index code for incrementing I
JC	Index code for decrementing I
KK1 (1 or 2)	Index code for a computed go to
ALPHA	First term of the right-hand side of equation 6
BETA	Second term of the right-hand side of equation 6
X1	Sum of ALPHA and BETA
SOLVE	See Solve subroutine
N3	Number of flow entries occurring after inflow ceases

Read Subroutines:

Symbol	Description
A(I1) (I1 = 1, K1)	List items of entries being read
I1	Index of implied do loop
K1	Number of items in list

Table Look-Up Subroutine:

Symbol	Description
X4	Look-up argument
A (J) (J = 1, N2)	Look-up table
J	Upper index of table bracket
K	Lower index of table bracket

Solve Subroutine:

Symbol	Description
A2(I)	Current area value for iteration
Q2(I)	Current flow value for iteration
X2	Left-hand side of equation 6
X2	Difference between X2 and X1
AU	Upper bound of area in iteration process
FAU	X2 if value of area too large
AL	Lower bound of area in iteration process
FAL	X2 if value of area too small
DELA	Area increment or decrement used to establish an upper or lower bound
ANEW	New value of area in iteration process
X3	Difference between new area value and previous area value
TOLR	Iteration cutoff

APPENDIX 1

MAIN LINE PROGRAM

```
C       STORAGE FLOOD ROUTING WITHOUT COEFFICIENTS                              001
C       ****************************************************************        002
        DIMENSION AREA1(50),DISCH1(50),Q1(100),A1(100),AREA2(50)                003
        DIMENSION DISCH2(50),Q2(100),A2(100),QL1(100),QL2(100)                  004
        COMMON AREA1,DISCH1,Q1,N1,I,J,K,M,AREA2,DISCH2,A2,Q2,X1,DELI,DELI1      005
        COMMON N2,X4,X2,DELA,DELA1,ANEW,TOLR,X3                                 006
        COMMON A1,DELX,QL1,QL2                                                  007
C       ****************************************************************        008
        XINT(C,D,E,F,G)=C+((G-E)/(F-E))*(D-C)                                   009
C       C=LOWER BOUND FOR WANTED TABLE                                         010
C       D=UPPER BOUND FOR WANTED TABLE                                         011
C       E=LOWER BOUND FOR ARGUMENT TABLE                                       012
C       F=UPPER BOUND FOR ARGUMENT TABLE                                       013
C       G=ARGUMENT                                                             014
C       ****************************************************************        015
C       READ PAR CARD                                                         016
        READ1,DELT,DELT1,DELA,DELA1,TOLR,N1,N2                                  017
      1 FORMAT(2F5.0,3F4.1,2I3)                                                 018
C       ****************************************************************        019
C       READ FIRST SECTION RATING AND INFLOW                                  020
        CALL READ (AREA1,N2)                                                    021
        CALL READ (DISCH1,N2)                                                   022
        CALL READ (Q1,N1)                                                       023
        CALL READ(QL1,N1)                                                       024
C       ****************************************************************        025
C       CALCULATE FIRST SECTION AREAS                                         026
     19 DO 10 I=1,N1                                                            027
        CALL TBLLP(DISCH1,Q1)                                                   028
     10 A1(I)=XINT(AREA1(K),AREA1(J),DISCH1(K),DISCH1(J),Q1(I))                 029
C       ****************************************************************        030
C       READ SECOND SECTION RATING AND LATERAL INFLOW                         031
        LL1=0                                                                   032
        LL2=0                                                                   033

        N11=N1                                                                  034
     59 M=N1                                                                    035
     99 I=1                                                                     036
        N1=N11                                                                  037
        J1=N1+1                                                                 038
        IF(M-N1) 952,952,951                                                    039
    951 DO 950 J=J1,M                                                           040
    950 QL2(J)=0.0                                                              041
    952 CONTINUE                                                                042
        CALL READ (AREA2,N2)                                                    043
        CALL READ(DISCH2,N2)                                                    044
        CALL READ(QL2,N1)                                                       045
C       ****************************************************************        046
C       READ SECOND SECTION INITIAL VALUE AND DELX                            047
        READ 2,DELX,Q2(I)                                                       048
      2 FORMAT (F5.0,F8.0)                                                      049
        CALL TBLLP(DISCH2,Q2)                                                   050
        A2(I)=XINT(AREA2(K),AREA2(J),DISCH2(K),DISCH2(J),Q2(I))                 051
        J=1                                                                     052
        N1=M                                                                    053
C       ****************************************************************        054
C       ROUTING DURING INFLOW                                                 055
    100 ALPHA=(A1(I)+A2(I))/2.                                                  056
        I=J+1                                                                   057
        BETA=(DELT/DELX)*Q1(I)+(-A1(I)+(DELT)*(QL1(I)+QL2(I)))/2.               058
```

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MAIN LINE PROGRAM--Continued

```
      X1=ALPHA+BETA                                                        059
      IF (X1)101,101,102                                                   060
  101 Q2(I)=Q2(1)                                                          061
      A2(I)=A2(1)                                                          062
      DELT=DELT+DELT1                                                      063
      I=I-J                                                                064
      J=J+1                                                                065
      IF(J-N1)100,105,105                                                  066
  105 I=J                                                                  067
      GO TO 111                                                            068
  102 CALL SOLVE                                                           069
      IF (Q2(I)-Q2(1)-.1)698,698,699                                       070
  698 Q2(I)=Q2(1)                                                          071
      A2(I)=A2(1)                                                          072
  699 CONTINUE                                                             073
      IF(SENSE SWITCH 2) 700,701                                           074
  700 PUNCH 702,A2(I),Q2(I),I                                              075
  702 FORMAT (2F13.3,I3)                                                   076
  701 CONTINUE                                                             077
      DELT=DELT1                                                           078
      IF (I-N1)110,111,111                                                 079
  110 J=I                                                                  080
      GO TO 100                                                            081
C     ****************************************************************     082
C     ROUTING AFTER INFLOW                                                 083
  111 N3=0                                                                 084
  112 J=I                                                                  085
      N3=N3+1                                                              086
  501 ALPHA=(A1(N1)+A2(I))/2.                                              087
      I=J+1                                                                088
      BETA=(DELT/DELX)*Q1(N1)+(-A1(N1)+DELT*(QL1(N1)+QL2(N1)))/2.          089
      X1=ALPHA+BETA                                                        090
      IF(X1)502,502,505                                                    091
  502 Q2(I)=Q2(1)                                                          092
      A2(I)=A2(1)                                                          093
      GO TO 600                                                            094
  505 CALL SOLVE                                                           095
      IF (Q2(I)-Q2(1)-.1)502,502,601                                       096
  601 GO TO 112                                                            097
  600 M=M+N3                                                               098
      IF (M-100)602,602,801                                                099
  801 M=100                                                                100
  602 CONTINUE                                                             101
      I2=N1+1                                                              102
      DO 900 I=I2,M                                                        103
      QL1(I)=0.0                                                           104
      QL2(I)=0.0                                                           105
      A1(I)=A1(N1)                                                         106
  900 Q1(I)=Q1(N1)                                                         107
      LL1=LL1+1                                                            108
      LL2=LL1+1                                                            109
C     ****************************************************************     110
C     PUNCH OUT AND INTERCHANGE                                            111
      PUNCH 4                                                              112
    4 FORMAT(///)                                                          113
      PUNCH 5,LL1,LL2                                                      114
    5 FORMAT(16H IN SECTION NO =I3,14X,17H OUT SECTION NO =I3)             115
      PUNCH 6                                                              116
```

MAIN LINE PROGRAM--Continued

```
    6 FORMAT(4X,8H IN AREA,6X,9H IN DISCH,10X,9H OUT AREA,5X,10H OUT DIS    117
   1CH)                                                                     118
    3 FORMAT (2F13.3,5X,2F13.3)                                             119
      DO 115 I=1,M                                                          120
      PUNCH 3,A1(I),Q1(I),A2(I),Q2(I)                                       121
      QL1(I)=QL2(I)                                                         122
      A1(I)=A2(I)                                                           123
  115 Q1(I)=Q2(I)                                                           124
C     ****************************************************************      125
C     INTERCHANGE RATING TABLES                                            126
      DO 116 I=1,N2                                                         127
      AREA1(I)=AREA2(I)                                                     128
  116 DISCH1(I)=DISCH2(I)                                                   129
      GO TO 99                                                              130
  800 END                                                                   131
```

Read and Table Look-up Subroutine

```
C     READ SUBROUTINE                                                       001
      SUBROUTINE READ(A,K1)                                                 002
      DIMENSION A(100)                                                      003
      DIMENSION AREA1(50),DISCH1(50),Q1(100),A1(100),AREA2(50)              004
      DIMENSION DISCH2(50),Q2(100),A2(100),QL1(100),QL2(100)                005
      COMMON AREA1,DISCH1,Q1,N1,I,J,K,M,AREA2,DISCH2,A2,Q2,X1,DELI,DELI1    006
      COMMON N2,X4,X2,DELA,DELA1,ANEW,TOLR,X3                               007
      COMMON A1,DELX,QL1,QL2                                                008
      READ 2,(A(I1),I1=1,K1)                                                009
    2 FORMAT (9F8.0)                                                        010
      RETURN                                                                011
      END                                                                   012


C     TABLE LOOK-UP SUBROUTINE                                              001
C     A=ARGUMENT TABLE                                                      002
C     B=ARGUMENT                                                            003
      SUBROUTINE TBLLP(A,B)                                                 004
      DIMENSION A(100),B(50)                                                005
      DIMENSION AREA1(50),DISCH1(50),Q1(100),A1(100),AREA2(50)              006
      DIMENSION DISCH2(50),Q2(100),A2(100),QL1(100),QL2(100)                007
      COMMON AREA1,DISCH1,Q1,N1,I,J,K,M,AREA2,DISCH2,A2,Q2,X1,DELT,DELT1    008
      COMMON N2,X4,X2,DELA,DELA1,ANEW,TOLR,X3                               009
      COMMON A1,DELX,QL1,QL2                                                010
      X4=B(I)                                                               011
      IF (X4)16,15,16                                                       012
   15 J=2                                                                   013
      GO TO 20                                                              014
   16 DO 30 J=1,N2                                                          015
      IF (A(J)-X4)30,20,20                                                  016
   20 K=J-1                                                                 017
      RETURN                                                                018
   30 CONTINUE                                                              019
      END                                                                   020
```

Solve Subroutine

```
C       SOLVE ROUTINE                                                       001
        SUBROUTINE SOLVE                                                    002
        DIMENSION AREA1(50),DISCH1(50),Q1(100),A1(100),AREA2(50)           003
        DIMENSION DISCH2(50),Q2(100),A2(100),QL1(100),QL2(100)             004
        COMMON AREA1,DISCH1,Q1,N1,I,J,K,M,AREA2,DISCH2,A2,Q2,X1,DELT,DELT1 005
        COMMON N2,X4,X2,DELA,DELA1,ANEW,TOLR,X3                            006
        COMMON A1,DELX,QL1,QL2                                             007
        XINT(C,D,E,F,G)=C+((G-E)/(F-E))*(D-C)                             008
        AU =0.                                                             009
        FAU =0.                                                            010
        AL=0.                                                              011
        FAL =0.                                                            012
        DELA =DELA1                                                        013
        IF(I-N1) 800,800,801                                               014
    801 A2(I)=A1(N1)                                                       015
        GO TO 11                                                           016
    800 A2(I)=A1(I)                                                        017
     11 CALL TBLLP(AREA2,A2)                                               018
        Q2(I)=XINT(DISCH2(K),DISCH2(J),AREA2(K),AREA2(J),A2(I))            019
        IF(SENSE SWITCH 1)700,701                                          020
    700 PUNCH 702,A2(I),Q2(I),I                                            021
    702 FORMAT (2F13.3,I3)                                                 022
    701 CONTINUE                                                           023
        X2=(DELT/DELX)*Q2(I)+(A2(I))/2.                                    024
        X2=X2-X1                                                           025
        IF (X2)3,5,2                                                       026
      2 DELA=DELA1                                                         027
        AU=A2(I)                                                           028
        FAU=X2                                                             029
        IF (AL)4,12,4                                                      030
     12 A2(I)=A2(I)-DELA                                                   031
     13 IF (A2(I))7,7,11                                                   032
      7 DELA=DELA*.5                                                       033
        A2(I)=A2(I)+DELA                                                   034
        GO TO 13                                                           035
      3 DELA=DELA1                                                         036
        AL=A2(I)                                                           037
        FAL=-X2                                                            038
        IF (AU)4,14,4                                                      039
     14 A2(I)=A2(I)+DELA                                                   040
     15 IF (A2(I)-AREA2(N2))11,11,16                                       041
     16 DELA=DELA*.5                                                       042
        A2(I)=A2(I)-DELA                                                   043
        GO TO 15                                                           044
      4 ANEW=AU-(FAU/(FAU+FAL))*(AU-AL)                                    045
        X3=ANEW-A2(I)                                                      046
        X3=ABSF(X3)                                                        047
        IF (X3-TOLR)5,5,10                                                 048
     10 A2(I)=ANEW                                                         049
        GO TO 11                                                           050
      5 A2(I)=ANEW                                                         051
        CALL TBLLP(AREA2,A2)                                               052
        Q2(I)=XINT(DISCH2(K),DISCH2(J),AREA2(K),AREA2(J),A2(I))            053
        RETURN                                                             054
        END                                                               055
```

APPENDIX 2

Program Input Format[1]

Parameter Card 1:

1–5	6–10	11–14	15–18	19–22	23–25	26–28
XXXXX	XXXXX	XXX.X	XXX.X	XXX.X	XXX	XXX
DELT	DELTI	DELA	DELA1	TOLR	N1	N2

Hydrograph and Rating Table Entries:

1–8	9–16	17–24	25–32	33–40	41–48
XXXXXXXX.	XXXXXXXX.	XXXXXXXX.	XXXXXXXX.	XXXXXXXX.	XXXXXXXX.

49–56	57–64	65–72	73–80[2]
XXXXXXXX.	XXXXXXXX.	XXXXXXXX.	XXXXXXXX.

Parameter Card 2:

1–5	6–13
XXXXX.	XXXXXXXX.

Program Output Format[1]

Inflow and Outflow Values:

1–13	14–26	32–44	45–57
XXXXXXXXXX.XXX	XXXXXXXXXX.XXX	XXXXXXXXXX.XXX	XXXXXXXXXX.XXX
A1(I)	Q1(I)	A2(I)	Q2(I)

Loading Sequence

1. Parameter card 1
2. Flow area portion of inflow section rating table
3. Flow portion of inflow section rating table
4. Inflow hydrographs at inflow section
5. Lateral inflow at inflow section
6. Flow area portion of outflow section rating table
7. Flow portion of outflow section rating table
8. Lateral inflow at outflow section
9. Parameter card 2

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[1] Decimal is shown at its implied position. If another position is required, it should be punched.

[2] Columns 73–80 are reserved for identification, such as card number, card type, section number, watershed location, etc.

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